REGULAR REFLECTION OF PLANE DETONATION WAVES FROM AN ELASTIC HALF-SPACE

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Abstract: An effective method for the approximate solution of the Eq. [1] for the intensity of a reflected shock wave in the case of oblique incidence of a detonation wave on an elastic half-space is described; the elastic half-space is described by a certain specific form of the equation of state. Formulas relating the front and particle velocities behind the transmitted wave front to physical parameters are derived. Values of the wave intensity and other quantities determined with the aid of a Ural-2 computer are cited.

The author of [1,2] investigated the regular reflection of shock waves from the boundary between two bodies. In the present paper we solve the analogous problem in the case of oblique incidence of a detonation wave on an elastic half-space. The detonation wave deforms the elastic half-space, which assumes the position OK_1 (Fig. 1) forming the angle β to the initial direction KO of the half-space boundary. We assume that the acoustic stiffness of the half-space is larger than the acoustic stiffness of the explosive. In this case, both reflected wave 2 and transmitted wave 3 are shock waves [3]. Let us denote the velocities of propagation of the detonation, reflected, and transmitted waves by $U_i (i = 1, 2, 3)$, respectively; let the pressure be p_i and let the density be $\rho_i (i = 0, 1, 2, 3, 4)$. The quantities U_1, α_1, ρ_0, and ρ_4 are given. We determine the intensities of waves 2 and 3, their velocities of propagation, and the angles α_2, α_3, and β. The parameters are constant within each of the domains a, b, c, d, and e. In domains a and e the medium is stationary, i.e., $u_0 = u_4 = 0$. The basic equations of the problem express the conditions at the wave fronts and the dynamic and kinematic relationships.

At the detonation wave front

$$u_1 = \frac{U_1}{k+1}, \qquad p_1 = \frac{\rho_0 U_1^2}{k+1},$$

$$a = \frac{kU_1}{k+1}, \qquad \rho_1 = \frac{k+1}{k}\rho_0,$$

where a and k are the speed of sound and the adiabatic exponent of the products of the explosion.

At the reflected wave front

$$(u_{1x} - u_{2x}) \cos \alpha_2 + (u_{1y} - u_{2y}) \sin \alpha_2 = u,$$

$$U_2 + u_{1x} \cos \alpha_2 + u_{1y} \sin \alpha_2 = U,$$

$$u_{1y} \cos \alpha_2 - u_{1x} \sin \alpha_2 = u_{2y} \cos \alpha_2 - u_{2x} \sin \alpha_2,$$

where [4]

$$u = \frac{a}{k}\left(\frac{p_2}{p_1} - 1\right)\left(1 + \frac{k+1}{2k}\,\frac{p_2 - p_1}{p_1}\right)^{-1/2},$$

$$U = a\left[1 + \frac{1}{2} k^{-1} p_1^{-1} (k+1)(p_2 - p_1)\right]^{1/2}.$$

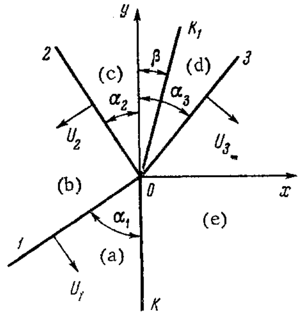

Fig. 1

At the refracted wave front

$$u_3 = u'(p_4, p_3), \qquad U_3 = U'(p_4, p_3).$$

Analytic expressions for the functions $U'(p_4, p_3)$ and $u'(p_4, p_3)$ are derived below. To close the system of equations we write out the self-evident kinematic relations

$$u_{1x} = u_1 \cos \alpha_1, \qquad u_{1y} = -u_1 \sin \alpha_1,$$

$$u_{3x} = u_3 \cos \alpha_3, \qquad u_{3y} = -u_3 \sin \alpha_3,$$

$$\frac{U_1}{\sin \alpha_1} = \frac{U_2}{\sin \alpha_2} = \frac{U_3}{\sin \alpha_3} = \frac{u_{2x} \cos \beta - u_{2y} \sin \beta}{\sin \beta},$$

$$u_{2x} \cos \beta - u_{2y} \sin \beta = u_{3x} \cos \beta - u_{3y} \sin \beta$$

and the dynamic relations

$$p_2 = p_3, \; p_0 = p_4.$$

Equations (1) express the absolute translational velocity of the point O.

Let us find the functions u' and U' in the case in which the elastic medium has an equation of state of the form

$$p = A_1 [(V_4 / V)^\gamma - 1] \qquad (V = 1/\rho;\ A_1,\ \gamma = \text{const}).$$

We assume that the pressure p is much larger than the pressure p_4. This equation affords a good description of the behavior of metals under pressures on the order of 100 000 atm [5].

Assuming that the process is adiabatic, we can write

$$dE = -p dV,$$

where E is the internal energy per unit mass of the medium. Integrating, we obtain

$$E_3 - E_4 = -A_1 \int_{V_4}^{V_3} (V_4^\gamma V^{-\gamma} - 1)\, dV =$$

$$= \frac{p_3 V_3 + A_1 \gamma (V_3 - V_4)}{\gamma - 1},$$

or recalling that

$$V_3 = 1/\rho_3, \quad V_4 = 1/\rho_4, \quad p_3 = p_2,$$

$$E_3 - E_4 = \frac{p_2 \rho_4 - A_1 \gamma (\rho_3 - \rho_4)}{\rho_3 \rho_4 (\gamma - 1)}.$$

Since [3]

$$u_3 = u'(p_4, p_3) = u'(p_0, p_2) = u'(p_2) = \left[\frac{p_2 (\rho_3 - \rho_4)}{\rho_3 \rho_4}\right]^{1/2},$$

$$U_3 = U'(p_4, p_3) = U'(p_2) = \left[\frac{p_2 \rho_3}{\rho_4 (\rho_3 - \rho_4)}\right]^{1/2},$$

it follows that

$$E_3 - E_4 = \frac{p_3}{2}(V_4 - V_3)\ \frac{p_2}{2}\,\frac{\rho_3 - \rho_4}{\rho_3 \rho_4}.$$

Hence,

$$\frac{p_2 \rho_4 - A_1 \gamma (\rho_3 - \rho_4)}{\gamma - 1} = \frac{p_2}{2} (\rho_3 - \rho_4).$$

This means that

$$\frac{\rho_3}{\rho_4} = \frac{(\gamma + 1) p_2 + 2A_1 \gamma}{(\gamma - 1) p_2 + 2A_1 \gamma},$$

so that u' and U' can be written as

$$u'(p_2) = \frac{p_2\sqrt{2}}{\sqrt{\rho_4[(\gamma+1)p_2+2A_1\gamma]}},$$

$$U'(p_2) = \sqrt{\frac{(\gamma+1)p_2+2A_1\gamma}{2\rho_4}}.$$

Expressing all the known quantities in terms of $\mu = p_2/p_1$, we obtain

$$U_2/U_1 = \sin\alpha_2/\sin\alpha_1 = f(\mu),$$
$$U_3/U_1 = \sin\alpha_3/\sin\alpha_1 = n(\mu),$$
$$\operatorname{ctg}\beta = \frac{U_1 - \sin^2\alpha_1[u_1 + uf(\mu)]}{u_1\cos\alpha_1 - u[1-f^2(\mu)\sin^2\alpha_1]^{1/2}},$$

where

$$n(\mu) = U_1^{-1}(p_1/2\rho_4)^{1/2}[(\gamma+1)\mu + 2A_1\gamma p_1^{-1}]^{1/2},$$
$$f(\mu) = CU - (B - AU^2)^{1/2},\quad A = u_1^2Q^{-2}\cos^2\alpha_1\sin^2\alpha_1,$$
$$B = u_1^2Q^{-1}\cos^2\alpha_1,\quad C = Q^{-1}(U_1 - u_1\sin^2\alpha_1),$$
$$Q = (U_1 - u_1\sin^2\alpha_1)^2 + u_1^2\cos^2\alpha_1\sin^2\alpha_1.$$

The intensity μ of the reflected shock wave can now be determined [1] from the equations

$$\left(\frac{U_1}{\sin^2\alpha_1} - \frac{p_1\mu}{\rho_4U_1}\right)\left[u_1\cos\alpha_1 - \frac{a\sqrt{2}(\mu-1)P(\mu)}{\sqrt{k(k-1)+k(k+1)\mu}}\right] =$$
$$= \left[\frac{U_1}{\sin^2\alpha_1} - u_1 - \frac{a\sqrt{2}(\mu-1)f(\mu)}{\sqrt{k(k-1)+k(k+1)\mu}}\right]\times$$
$$\times\frac{(2p_1/\rho_4)^{1/2}\mu\Phi(\mu)}{[(\gamma+1)\mu+2A_1\gamma p_1^{-1}]^{1/2}}, \qquad (2)$$

$$\mu_{0i} = \Big[5k + 2\sqrt{2}\varphi_{i-1} + 1 +$$
$$+\sqrt{17k^2+2k+1+4\varphi_{i-1}k^{-1}[(k-1)\varphi_{i+1}+k\sqrt{2}(3k-1)]}\Big]\times$$
$$\times\left[4(k+\sqrt{2}\varphi_{i-1}+{}^1/_2\,k^{-1}\varphi_{i-1}^2)\right]^{-1}, \qquad (3)$$

where

$$P(\mu) = [1 - f^2(\mu)\sin^2\alpha_1]^{1/2},\qquad \Phi(\mu) = [1 - n^2(\mu)\sin^2\alpha_1]^{1/2},$$
$$\varphi_{i-1} = (2p_1/\rho_4)^{1/2}(k+1)U_1^{-1}\sqrt{\frac{k(k-1)+k(k+1)\mu_{0,\,i-1}}{(\gamma+1)\mu_{0,\,i-1}+2A_1\gamma p_1^{-1}}}.$$

Table 1

α_1	μ	α_2	α_3	10 β	U_3/U_1
0.17	1.82	0.12	0.16	0.83	0.69
0.35	1.81	0.25	0.31	0.80	0.72
0.52	1.79	0.40	0.47	0.76	0.78
0.66	1.78	0.54	0.59	0.71	0.84
0.70	1.78	0.58	0.62	0.70	0.85
0.73	1.79	0.62	0.65	0.68	0.87
0.77	1.79	0.67	0.68	0.67	0.89
0.80	1.80	0.72	0.70	0.66	0.92
0.84	1.81	0.77	0.73	0.65	0.94
0.87	1.84	0.84	0.76	0.64	0.97
0.91	1.89	0.92	0.79	0.64	1.01

Table 2

α_1	μ	α_2	α_3	β	U_3/U_1
0.17	1.43	0.11	0.20	0.15	0.61
0.35	1.44	0.22	0.39	0.15	0.64
0.52	1.45	0.35	0.59	0.13	0.69
0.66	1.46	0.48	0.76	0.12	0.76
0.70	1.47	0.52	0.81	0.12	0.77
0.73	1.48	0.56	0.85	0.11	0.79
0.77	1.49	0.60	0.90	0.11	0.81
0.80	1.51	0.65	0.95	0.10	0.84
0.84	1.53	0.70	1.00	0.10	0.86
0.87	1.57	0.75	1.06	0.09	0.89
0.91	1.62	0.83	1.12	0.08	0.93

Calculations on the basis of (3) show that the first approximation already yields results which agree with the data of [3]. In the case of an absolutely rigid barrier $\varphi \equiv 0$ and

$$\mu_0 = {}^1/_4\,k^{-1}[5k + 1 + (17k^2 + 2k + 1)]^{1/2}.$$

We used the above method to calculate the quantities μ, α_2, α_3, β, U_2/U_1, U_3/U_1, and ρ_3/ρ_4 on a Ural-2 computer for two cases:

1) $A_1 = 4.41 \times 10^{10}$ N/m^2; $\rho_4 = 7.81 \times 10^3$ kg/m^3;

2) $A_1 = 2.00 \times 10^{10}$ N/m^2; $\rho_4 = 2.70 \times 10^3$ kg/m^3.

The quantities ρ_0, U_1, k, and γ were set equal to ρ_0 $1.30 \cdot 10^3$ kg/m^3, $U_1 = 6020$ m/sec, $k = 3$, and $\gamma = 4$. We need go no further than three approximations to obtain the intensity μ of the reflected shock wave within three places. The results for the above cases appear in Tables 1 and 2.

Within the required error bracket we have

$$0.90 \leqslant U_3/U_1 \leqslant 0.91,\quad 0.083 \leqslant u_3/U_1 \leqslant 0.087,$$
$$\frac{\rho_3}{\rho_4} = 1.10$$

in the first case and

$$1.12 \leqslant U_3/U_1 \leqslant 1.14,$$
$$0.15 \leqslant u_3/U_1 \leqslant 0.17,\quad 1.16 \leqslant \rho_3/\rho_4 \leqslant 1.18$$

in the second.

Our data indicate that the intensity of the reflected shock wave diminishes with decreasing density of the elastic medium. In the above cases, the ranges $\alpha_1 > 53°$ and $\alpha_1 > 54°$, respectively, are characterized by irregular reflection for which the above equations are invalid.

REFERENCES

1. A. I. Gubanov, "Reflection and refraction of shock waves at the boundary between two media, II. Regular reflection with oblique incidence," ZhTF, vol. 28, p. 615, 1959.

2. A. I. Gubanov, "Reflection and refraction of shock waves at the boundary between two media, I. The case of normal incidence," ZhTF, Vol. 29, p. 2035, 1958.

3. F. A. Baum, K. P. Stanyukovich, and B. I. Shekhter, The Physics of Explosions [in Russian], Moscow, 1959.

4. L. G. Loitsyanskii, Mechanics of Liquids and Gases [in Russian], Moscow-Leningrad, 1950.

5. Ya. B. Zel'dovich and Yu. P. Raizer, The Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena [in Russian], Moscow, 1963.

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